MR2004588 (2004m:01001) 01-06

★**Word, image, number.**

Communication in the Middle Ages.
Selected papers from the conference held at Pennsylvania State University, University Park, PA, October 4–5, 1997.
Edited by John J. Contreni and Santa Casciani.
Micrologus' Library, 8.
SISMEL—Edizioni del Galluzzo, Florence, 2002. 457 *pp.* €65.00. *ISBN* 88-8450-021-4

The 14 papers in this collection include the following, which will not be reviewed individually: John J. Contreni, Counting, calendars, and cosmology: numeracy in the early Middle Ages (43–83) MR2004589; Wesley M. Stevens, Fields and streams: language and practice of arithmetic and geometry in early medieval schools (113–204) MR2004590; Marie-Thérèse Zenner, Imaging a building: Latin Euclid and practical geometry (219–246) MR2004591; J. Lennart Berggren, Medieval arithmetic: Arabic texts and European motivations (351–365) MR2004592; Frank J. Swetz, Figura mercantesco: merchants and the evolution of a number concept in the latter Middle Ages (391–412) MR2004593.

Seven of the papers in the volume were presented at a conference organized by the Center for Medieval Studies at The Pennsylvania State University in 1997. The remaining seven were invited for the volume. Not all have an evident relation to the topic of medieval communication, and such obvious topics as "correspondences" and "diplomacy" are absent. The interest of the volume thus consists in the sum of the interest of the contributions taken singly.

Five of these, listed above, have to do with the history of mathematics:

John Contreni's contribution covers not only what most authors would speak of as "early Middle Ages" (until Charlemagne) but goes until c. 1100. Its claim is that European learning was *not* restricted to "the compends of Isidore and Bede and scattered fragments of Roman learning", as Ch. H. Haskins stated in 1927, and that it was highly numerate in many respects—not least around the sophisticated discipline of *computus*, calendar reckoning. This claim is well supported. The article reveals no profound understanding of the basic mathematical subject matter; for instance, the author seems to believe that "Arabic" numeration is thought superior to Roman ditto because of the shape of the digits, not because of its place value system; *summa*, further, is translated "sum" even when the *summa*, i.e., the "amount" of a single number is meant. For the purpose of the article this is not very important, and the rich footnotes can be recommended as containing references to much recent work of high quality.

Wesley Stevens' paper does not treat much of what is stated in the title. Instead it presents us with a thorough survey of how (indeed, how badly) extant lexicographic tools—all the most respected dictionaries for Latin in general or for Medieval Latin in particular—cover the uses of a large number of words as mathematical technical terms (*abacus, ablatio, addo, adverbialis*, . . . —88 in total, if I counted well). The "language and practice of arithmetic and geometry in early medieval schools" thus only appears when Stevens shows that the missing mathematical uses are in fact documented in the sources. Stevens, one might say, demonstrates how difficult it is for modern scholars (producers and users of dictionaries) to communicate with the Middle Ages.

Marie-Thérèse Zenner's contribution "presents current findings from a long-term project designed to investigate documents in the mathematical sciences for ideas in design and construction during the Romanesque period". Much of the paper is meant to disprove Ron Shelby's claim that the masons' geometry was "not Euclidean geometry; there are no axioms or proofs, and almost no mathematical formulas or calculations were involved in the work of the masons". Unfortunately, she identifies "Euclidean geometry" with what is found in the so-called Boethius-I and Boethius-II geometries and in various agrimensor writings (certainly not what Shelby intends), and believes that if the correctness of a simple mason's construction *can* be proved from a proposition in the *Elements*, then the *Elements* must necessarily be the source for that construction. This *petitio principii* turns up repeatedly. The arguments against Shelby and in favour of Euclidean influence thus boil down to nothing.

A second part of the article presents Zenner's own measurements of the St.-Étienne church in Nevers, where she seems to have found good evidence for a construction based on circles (in no need of Euclid). Further mathematical speculations on this construction are problematic: Three linear extensions of 54, 64 and 70 units (not located together in the plan) are supposed to represent the golden section because 64 divides the interval 54–70 in the ratio 3:5. The possible use of an irregular five-pointed star in the layout is taken as further evidence for the importance of this ratio.

Lennart Berggren's short article deals first with the many number systems in use during the European Middle Ages: Roman numerals; a simplified version of these; the "Basingstoke–Cistercian" notation; numbers represented by the fingers (used in particular for retaining partial results during mental calculation); the abacus with simple counters; the one with marked counters (the "monastic abacus" ascribed by Berggren without hesitation to Gerbert); and then, finally, in the second part of the article, the Arabic numerals, introduced first for use in astronomical calculation (the later use in commercial arithmetic is left out, since it is dealt with in Swetz's article). The third part of the article presents the use of sexagesimal fractions (occasionally in the fourteenth century, of fully sexagesimal numbers). A small correction (p. 357): Leonardo Fibonacci does not state in the *Liber abbaci* that he had been taught on a Gerbert abacus in his youth, but that his father took him to Bejaïa in present-day Algeria, where he dedicated himself to *studio abbaci* for "some days"—certainly no monastic abacus but computation (with Hindu-Arabic numerals).

Frank Swetz gives as fair a coverage of the Italian abbacus tradition and its northern descendants as can be done without taking into account anything published by Italian, French, Spanish or German scholars on the matter (in their own tongue or in English) except to the extent it can be cited indirectly from anglophone sources. The thesis—that the abbacus environment furthered the emergence of a new number concept—confronts the numbers of commercial arithmetic, which can be operated upon, to the "mystical" numbers of the Pythagorean-Platonic kind—those to which "no theorem applies" unless one tries to invent theorems not based in mathematics, as sarcastically observed by Aristotle (*Metaphysics* N, 1090b27ff). The latter were certainly very popular among medieval scholars, and more so than theoretical arithmetic in the style of *Elements* VII–IX (which, however, was not as totally absent as one might believe from the article). The thesis seems reasonable in itself but is not supported by specific arguments—that late medieval texts contain numbers that indicate measures and can be operated upon does not distinguish them from the numbers used in Carolingian and later *computus* (cf. Contreni's article). A minor correction: the *regula del chataina* is not, as stated by Swetz (p. 409), the "chain rule" (composite rule of three) but the rule of a "double false position".

Reviewed by *Jens Høyrup*